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Furthermore, adding  $AO$  to  $OK$  we find

$$AK = \frac{(AO + AO')FO''}{AO - AO'} = \frac{(2R' + R'')r}{R''} = \frac{R''(R'' + 15r')^{\frac{1}{2}}(R'' + 15r' - 1)}{R'' + 15r' + 3},$$

and, similarly,

$$AM = \frac{(AO + AO')EQ'}{AO - AO'} = \frac{(2R' + R'')r'}{R''} = r'\sqrt{(R'' + 15r')}. \quad \text{Hence this}$$

*Construction.*—On the indefinite straight line  $AX$ , set off

$$AB = 2AO' = \frac{1}{2}R''[\sqrt{(R'' + 15r')} - 1],$$

$$AD = 2AO = \frac{1}{2}R''[\sqrt{(R'' + 15r')} + 1],$$

$$BQ = R''.$$

Bisect  $AB$  in  $O'$ , and  $AD$  in  $O$ : then, with centers  $O'$ ,  $O$  and  $Q$ , and with radii  $AO'$ ,  $AO$  and  $BQ$ , describe the circles  $AEB$ ,  $AHLD$  and  $BGD$ . These will obviously touch each other, two and two.

Again, on  $AD$  set off

$$AK = \frac{R''\sqrt{(R'' + 15r')(R'' + 15r' - 1)}}{R'' + 15r' + 3},$$

and

$$\begin{aligned} AM &= r'\sqrt{(R'' + 15r')}. \quad \text{At } K, \text{ erect the perpend. } KO'' \\ &= \frac{2R''(R'' + 15r' - 1)}{R'' + 15r' + 3}, \end{aligned}$$

and at  $M$  erect the perpendicular  $MQ' = 4r'$ . Finally, with centers  $O''$  and  $Q'$ , and radii  $FO'' = R''(R'' + 15r' - 1) \div (R'' + 15r' + 3)$  and  $EQ' = 4r'$ , describe the circles  $FIL$ ,  $EIH$  and it is done.

[The foregoing solution, it will be seen, applies only to a special case of the question quoted at p. 143, Vol. V, as the relative position of the given circles cannot be arbitrarily taken. No general solution has been rec'd.]

## NOTE ON THE DIFFERENTIATION OF EXPONENTIAL AND TRIGONOMETRICAL FUNCTIONS.

BY ARTEMAS MARTIN, M. A., ERIE, PA.

THE following methods may not be new, but I have not met with them in any of the many works on the calculus to which I have access.

1. Differentiate  $y = a^x$ .

We have, by the Exponential Theorem,

$$y = a^x = 1 + x \log a + \frac{x^2(\log a)^2}{1.2} + \frac{x^3(\log a)^3}{1.2.3} + \text{etc};$$

$$\begin{aligned}\therefore dy &= d(a^x) = (\log a)dx + x(\log a)^2dx + \frac{x^2(\log a)^3dx}{1 \cdot 2} + \text{etc.}, \\ &= \log a \left[ 1 + x \log a + \frac{x^2(\log a)^2}{1 \cdot 2} + \frac{x^3(\log a)^3}{1 \cdot 2 \cdot 3} + \text{etc.} \right] dx, \\ &= a^x \log a dx.\end{aligned}$$

2. Differentiate  $y = \sin x$ .

We have, by Analytical Trigonometry,

$$\begin{aligned}y = \sin x &= x - \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \frac{x^7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{etc.}, \\ \cos x &= 1 - \frac{x^2}{1 \cdot 2} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{x^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}; \\ \therefore dy &= d(\sin x) = dx - \frac{x^2 dx}{1 \cdot 2} + \frac{x^4 dx}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{x^6 dx}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}, \\ &= \left[ 1 - \frac{x^2}{1 \cdot 2} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{x^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.} \right] dx, \\ &= \cos x dx.\end{aligned}$$


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EVALUATION OF THE EULERIAN INTEGRAL  $u = \int_0^{\frac{\pi}{2}} \log(\sin x) dx$ , BY  
PROF. E. J. EDMUNDS, NEW ORLEANS, LA.—We have

$$u = \lim_{n \rightarrow \infty} \frac{\pi}{2n} \left[ \log \sin \left( \frac{1}{n} \cdot \frac{\pi}{2} \right) + \log \sin \left( \frac{2}{n} \cdot \frac{\pi}{2} \right) - \dots + \log \sin \left( \frac{n-1}{n} \cdot \frac{\pi}{2} \right) \right];$$

hence,

$$u = \lim_{n \rightarrow \infty} \frac{\pi}{2n} \cdot \log \left[ \sin \left( \frac{1}{n} \cdot \frac{\pi}{2} \right) \sin \left( \frac{2}{n} \cdot \frac{\pi}{2} \right) \dots \sin \left( \frac{n-1}{n} \cdot \frac{\pi}{2} \right) \right],$$

$n$  increasing to infinity.

But by Cotes' Theorem we have

$$\frac{z^{2n}-1}{z^2-1} = \left( z^2 - 2z \cos \frac{\pi}{n} + 1 \right) \left( z^2 - 2z \cos \frac{2\pi}{n} + 1 \right) \dots \left( z^2 - 2z \cos \frac{(n-1)\pi}{n} + 1 \right).$$

For  $z = 1$ , this equation becomes,

$$n = 2^{2(n-1)} \sin^2 \left( \frac{1}{n} \cdot \frac{\pi}{2} \right) \sin^2 \left( \frac{2}{n} \cdot \frac{\pi}{2} \right) \dots \sin^2 \left( \frac{n-1}{n} \cdot \frac{\pi}{2} \right); \text{ therefore}$$

$$\frac{\log n - 2(n-1) \log 2}{2} = \log \sin \left( \frac{1}{n} \cdot \frac{\pi}{2} \right) \dots \log \sin \left( \frac{n-1}{n} \cdot \frac{\pi}{2} \right).$$

Multiplying both members by  $\pi \div 2n$  and passing to limits we finally obtain

$$u = \frac{1}{2} \pi \cdot \log \frac{1}{2},$$

which is the value of the integral sought.